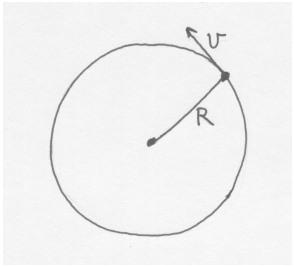
GFD I, Problem Set #2

Handed out 1/23/2012, Due at the start of class 1/30/2012

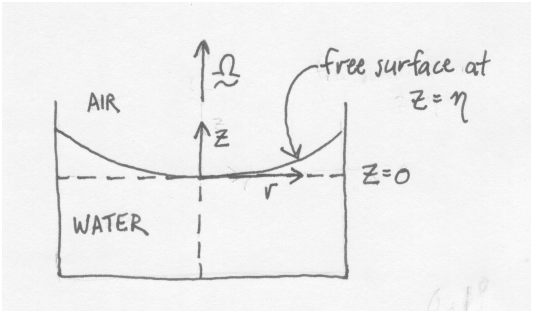
1) Consider a mass spinning around at the end of a string (ignore gravity) so that it has a circular path through space in the "fixed" frame of reference. It has constant speed v, and the string has length R, so this is the radius of the circle. In standard physics textbooks you learn that the centripetal acceleration has magnitude v^2/R, and is directed toward the center of the circle. Now consider the same motion as viewed from a rotating frame of reference. In this case we will stipulate that the angular rate of rotation of the frame of reference has magnitude Ω, which is <u>somewhat different</u> from v/R, so the mass is not stationary in the rotating frame of reference. Is the expression derived in class:



$$\frac{d\mathbf{u}_f}{dt} = \frac{d\mathbf{u}_r}{dt} + 2\boldsymbol{\Omega}\times\mathbf{u}_r - \Omega^2\mathbf{R}$$

consistent with the simple expression v^2/R? (Notation: I am using boldface in the equation to signify a vector, as in most textbooks).

2) Say you have a cylindrical bowl (1 m in radius) full of water of constant density, and it is spinning around on a turntable with a period of 2π s. Assume that the water has come to equilibrium, meaning that it is spinning at the same speed as the bowl ("solid body rotation"). What is the primary force balance in the vertical direction? What is the primary force balance in the radial direction? [Hint: these are most easily argued from the "fixed" frame of reference. See Kundu & Cohen Appendix B2 if you need help.] Using the answers to the above two questions, what is the mathematical form of the shape of the free surface of the water? What is the difference in surface height between the edge of the bowl and the center?



3) Calculate the difference in the magnitude of gravity (due to the centrifugal acceleration) between the equator and the pole on Earth. Express your answer in terms of a fraction of our "standard" value $g = 9.8$ m s^{-2}. At 45° N latitude how different is the direction we define as "down" (which includes the centrifugal acceleration) from the direction which points to the center of the Earth? Express you answer in degrees.